Tunable pure spin currents in a triple-quantum-dot ring

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Electron transport properties in a triple-quantum-dot ring with three terminals are theoretically studied. By introducing local Rashba spin-orbit interaction on an individual quantum dot, we calculate the charge and spin currents in one lead. We find that a pure spin current appears in the absence of a magnetic field. The polarization direction of the spin current can be inverted by altering the bias voltage. In addition, by tuning the magnetic field strength, the charge and spin currents reach their respective peaks alternately.

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One of the central issue in spintronics is how to realize the spin accumulation and spin transport in nanodevices. Recently, there has been many theoretical proposals to achieve the pure spin current without an accompanying charge current in mesoscopic systems, such as the use of spin Hall effects,[1,2] optical spin orientation by linearly polarized light,[3,4] adiabatic or nonadiabatic spin pumping in metals and semiconductors,[5,6] generation in three-terminal spin devices.[7] Among these schemes, spin-orbit(SO) coupling is exploited to influences the electron spin state. In particular, in low-dimensional structures Rashba SO interaction comes into play by introducing an electric potential to destroy the symmetry of space inversion in an arbitrary spatial direction.[8,9] Thus, by virtue of the Rashba interaction, electric control and manipulation of the electronic spin state is feasible.[10,11,12,13,14]

In this Letter, we introduce Rashba interaction to act locally on one component quantum dot(QD) of a triple-QD ring with three terminals. Our theoretical investigation indicates that it is possible to form the pure spin current in one of the three leads even in the absence of a magnetic field. And the polarization direction of the spin current can be inverted by altering the bias voltage.

The structure that we consider is illustrated in Fig.1. The single-particle Hamiltonian for an electron in such a structure can be written as $H_s = H_0 + H_{so} = \frac{\mathbf{P}^2}{2m^*} + V(\mathbf{r}) + H_{so}$ where, accompanying the kinetic energy term $\frac{\mathbf{P}^2}{2m^*}$, the electron confined potential $V(\mathbf{r})$ defines the structure geometry; And $H_{so} = \frac{\hat{y}}{2\hbar}\cdot[\alpha(\hat{\sigma}\times\mathbf{p}) + (\hat{\sigma}\times\mathbf{p})\alpha]$ denotes the local Rashba SO coupling on QD-2 (QD-j represents the QD with a single-particle level ε_j shown in Fig.1(a)). We select the basis set $\{\psi_{k_j}\chi_\sigma, \psi_j\chi_\sigma\}$($j$=1,2,3) to second-quantize the Hamiltonian. The wavefunctions ψ_j and ψ_{k_j} have the physical meaning of the orbital eigenstates of the isolated QD and leads, in the absence of Rashba interaction, where k_j indicates the continuum state in lead-j. χ_σ with $\sigma = \uparrow, \downarrow$ denotes the eigenstates of Pauli spin operator $\hat{\sigma}_z$.

The second-quantized Hamiltonian consists of three parts: $\mathcal{H}_s = \mathcal{H}_c + \mathcal{H}_d + \mathcal{H}_t$.

$$\begin{aligned}
\mathcal{H}_c &= \sum_{\sigma,k_j}\varepsilon_{k_j} c^\dagger_{k_j\sigma} c_{k_j\sigma}, \\
\mathcal{H}_d &= \sum_{j=1,\sigma}^{3}\varepsilon_j d^\dagger_{j\sigma} d_{j\sigma} + \sum_{l=1,\sigma}^{2}[t_{l\sigma} d^\dagger_{l\sigma} d_{l+1\sigma} + r_l(d^\dagger_{l\downarrow} d_{l+1\uparrow} \\
&\quad - d^\dagger_{l+1\downarrow} d_{l\uparrow})] + t_3 e^{i\phi} d^\dagger_{3\sigma} d_{1\sigma} + \text{H.c.} \\
\mathcal{H}_t &= \sum_{\sigma k_j} V_{j\sigma} d^\dagger_{j\sigma} c_{k_j\sigma} + \text{H.c.},
\end{aligned} \tag{1}$$

where $c^\dagger_{k_j\sigma}$ and $d^\dagger_{j\sigma}$ ($c_{k_j\sigma}$ and $d_{j\sigma}$) are the creation (annihilation) operators corresponding to the basis states in lead-j and QD-j. ε_{k_j} is the single-particle level in lead-j. $V_{j\sigma} = \langle\psi_j\chi_\sigma|H_s|\psi_{k_j}\chi_\sigma\rangle$ denotes QD-lead hopping amplitude. The interdot hopping amplitude, written as $t_{l\sigma} = t_l - i\sigma s_l (l = 1, 2)$, has two contributions: $t_l = \langle\psi_l|H_0|\psi_{l+1}\rangle$ is the ordinary transfer integral irrelevant to the Rashba interaction; $s_l = i\langle\psi_l|\alpha p_x + p_x\alpha|\psi_{l+1}\rangle$ indicates the strength of spin precession. In addition, the interdot spin flip term has the strength $r_l = \langle\psi_l|\alpha p_z + p_z\alpha|\psi_{l+1}\rangle$. A magnetic field penetrating the ring is described by a geometric phase factor ϕ.

Without loss of generality, we assume that each QD confines the electron by an isotropic harmonic potential $\frac{1}{2}m^*\omega_0 r^2$; and the three QDs are positioned on a circle equidistantly. Then by a straightforward derivation, we find some rough relationships between the relevant parameters in the above Hamiltonian: $t_1 = t_2$, $s_1 = s_2$, $r_1 = -r_2$, and $|s_l| = |r_l| \simeq \tilde{\alpha}t_l$, where $\tilde{\alpha} = \alpha\sqrt{m^*}/(3\sqrt{\hbar^3\omega_0})$, is a dimensionless Rashba coefficient. Following

these relations we can rewrite the interdot hopping amplitude in an alternative form: $t_{l\sigma} = t_l\sqrt{1+\tilde{\alpha}^2}e^{-i\sigma\varphi} = t_0 e^{-i\sigma\varphi}$ with $\varphi = \tan^{-1}\tilde{\alpha}$. Thus, just three independent parameters, t_0, $\tilde{\alpha}$ and the magnetic phase factor ϕ, are needed to characterize the interdot hopping. It should be noted that the Rashba interaction brings about a spin dependent phase factor $\sigma\varphi$.

We now proceed to study the electronic transport through this QD ring. By means of the Green function technique, at zero temperature the electron current with a specific spin in an arbitrary lead, say lead-1, can be expressed as[15]

$$J_{1\sigma} = \frac{e}{h}\sum_{j'\sigma'}\int_{\mu_1}^{\mu_{j'}} d\omega T_{1\sigma,j'\sigma'}(\omega), \tag{2}$$

where $T_{1\sigma,j'\sigma'}(\omega) = \Gamma_1 G^r_{1\sigma,j'\sigma'}(\omega)\Gamma_{j'}G^a_{j'\sigma',1\sigma}(\omega)$ denotes the transmission probability between spin-σ' electron in lead-j' and spin-σ electron in lead-1. $\Gamma_j = 2\pi|V_{j\sigma}|^2\rho_j(\omega)$, associated with the density of states in lead-j $\rho_j(\omega)$, can be usually regarded as a constant if ρ_j is a slow-varying function in the energy scale as far as the electron transport is concerned.[16] G^r and G^a, the retarded and advanced Green functions, are 6×6 matrixes for the triple-QD ring. They have the relationship $[G^r] = [G^a]^\dagger$. From the equation-of-motion method, we obtain the retarded Green function matrix,

$$[G^r]^{-1} = \begin{bmatrix} g_1^{-1} & -t_{1\uparrow} & -t_3 e^{-i\phi} & 0 & r_1^* & 0 \\ -t_{1\uparrow}^* & g_2^{-1} & -t_{2\uparrow} & -r_1^* & 0 & r_2^* \\ -t_3 e^{i\phi} & -t_{2\uparrow}^* & g_3^{-1} & 0 & -r_2^* & 0 \\ 0 & -r_1 & 0 & g_1^{-1} & -t_{1\downarrow} & -t_3 e^{-i\phi} \\ r_1 & 0 & -r_2 & -t_{1\downarrow}^* & g_2^{-1} & -t_{2\downarrow} \\ 0 & r_2 & 0 & -t_3 e^{i\phi} & -t_{2\downarrow}^* & g_3^{-1} \end{bmatrix}$$

In the above expression, g_j is the Green function of QD-j unperturbed by the other QDs and in the absence of Rashba effect. $g_j = [(z-\varepsilon_j) + \frac{i}{2}\Gamma_j]^{-1}$ with $z = \omega + i0^+$.

As for the chemical potentials in the three leads, we fix $\mu_1 = 0$. It is the reference point of energy of the system. And we let $\mu_2 = -\mu_3 = eV/2$ with V being a small bias voltage. Then the net charge J_{1c} and spin currents J_{1s} in lead-1 are respectively defined as $J_{1c} = J_{1\uparrow} + J_{1\downarrow}$ and $J_{1s} = J_{1\uparrow} - J_{1\downarrow}$.

Now we are ready to carry out the numerical calculation about the spectra of the charge and spin currents in lead-1. To do this, we choose the Rashba coefficient $\tilde{\alpha} = 0.5$ which is available under the current experimental circumstance[17]. And the bias voltage is $eV = 2t_0$ with t_0 being an appropriate unit of energy. In Fig.1(b) J_{1c} and J_{1s} versus the magnetic phase factor ϕ are shown. Besides, their traces as a function of the QD level are shown in Fig.1(c) and (d). The following interesting features in these spectra are noteworthy. With the variation of the applied magnetic field, the charge and spin currents oscillate out of phase. In the vicinity of $\phi = (n+\frac{1}{2})\pi$, namely, the magnetic phase factor is the odd multiple of $\pi/2$, J_{1c} reaches its maximum. Simultaneously, the spin current J_{1s} just be very close to a zero point. On the contrary, when $\phi = n\pi$ the situation is just inverted, the maximum of J_{1s} encounters the zero of J_{1c}. This indicates that a striking pure spin current can be implemented without an accompanying charge current. In particular, such a pure spin current emerges even at the vicinity of $\phi = 0$, which implies that an applied magnetic field is not an indispensable condition for the occurrence of the pure spin current.

In Fig.1(c), the currents versus the QD levels are shown in the absence of magnetic field. Apart from the pure spin current at a specific value of ε_0, one can find the more interesting phenomenon that the polarization direction of the spin current can be inverted by the reversal of the bias. In addition, as shown in Fig.1(d), when the coupling of the QD ring to lead-3 is cut off, the spin current disappears, though the Rashba interaction still exists. This means that the three-terminal configuration is a necessary condition for the occurrence of pure spin current.

The calculated transmission functions are plotted in Fig.2. They are just the integrands for the calculation of the charge and spin currents, see Eq.(2). By comparing the results shown in Fig.2(a) and (b) we can see that these transmission functions depend nontrivially on the magnetic phase factor. At $\phi = 0$, i.e., the zero magnetic field case, the traces of transmission functions $T_{1\sigma,2\sigma}(\omega)$ and $T_{1\bar{\sigma},3\bar{\sigma}}(\omega)$ coincide with each other very well. However, as shown in Fig.2(b), at $\phi = \pi/2$ the four transmission function show distinct traces. In this case a noticeable feature is the transmission between lead-1 and lead-2 is relatively suppressed, in comparison with $T_{1\sigma,3\sigma}(\omega)$. Substituting such integrands into the current formulae, and noting the opposite bias voltages of lead-2 and lead-3 with respect to lead-1, one can certainly arrive at the results of the distinct charge and spin currents at $\phi = 0$ and $\pi/2$ respectively, as shown in Fig.1.

The underlying physics being responsible for the spin dependence of the transmission functions is the quantum interference, which becomes manifest if we analyze the electron transmission process in language

of the Feynman path. First of all, we notice that the spin flip arising from the Rashba interaction does not play a leading role in causing the tunable spin and charge currents. To illustrate this, we plot the spectra of the charge and spin currents in the absence of the spin flip term(i.e., $r_l = 0$) in Fig.1(b). We can see that the corresponding results coincide with the complete spectra very well. Therefore, to keep the argument simple, we drop the spin flip term for the analysis of quantum interference.

We write $T_{1\sigma,2\sigma} = |\tau_{1\sigma,2\sigma}|^2$ by introducing the transmission probability amplitude which is defined as $\tau_{1\sigma,2\sigma} = \widetilde{V}_{1\sigma}^* G_{1\sigma,2\sigma}^r \widetilde{V}_{2\sigma}$ with $\widetilde{V}_{j\sigma} = V_{j\sigma}\sqrt{2\pi\rho_j(\omega)}$. The transmission probability amplitude $\tau_{1\sigma,2\sigma}$ can be divided into two terms, i.e., $\tau_{1\sigma,2\sigma} = \tau_{1\sigma,2\sigma}^{(1)} + \tau_{1\sigma,2\sigma}^{(2)}$, where $\tau_{1\sigma,2\sigma}^{(1)} = \frac{1}{D}\widetilde{V}_{1\sigma}^* g_1 t_{1\sigma} g_2 \widetilde{V}_{2\sigma}$ and $\tau_{1\sigma,2\sigma}^{(2)} = \frac{1}{D}\widetilde{V}_{1\sigma}^* g_1 t_3 e^{-i\phi} g_3 t_{2\sigma}^* g_2 \widetilde{V}_{2\sigma}$ with $D = \det\{[G^r]^{-1}\}$. By observing the structures of $\tau_{1\sigma,2\sigma}^{(1)}$ and $\tau_{1\sigma,2\sigma}^{(2)}$, we can readily recognize that they just represent the two paths from lead-2 to lead-1 via the two arms of the QD ring. The phase difference between them is $\Delta\phi_{2\sigma} = [\phi - 2\sigma\varphi - \theta_3]$ with θ_j being the argument of g_j. $T_{1\sigma,3\sigma}$ can be analyzed in a similar way. That is $T_{1\sigma,3\sigma} = |\tau_{1\sigma,3\sigma}^{(1)} + \tau_{1\sigma,3\sigma}^{(2)}|^2$, with $\tau_{1\sigma,3\sigma}^{(1)} = \frac{1}{D}\widetilde{V}_{1\sigma}^* g_1 t_3 e^{-i\phi} g_3 \widetilde{V}_{3\sigma}$ and $\tau_{1\sigma,3\sigma}^{(2)} = \frac{1}{D}\widetilde{V}_{1\sigma}^* g_1 t_{1\sigma} g_2 t_{2\sigma} g_3 \widetilde{V}_{3\sigma}$. The phase difference between the two paths is $\Delta\phi_{3\sigma} = [\phi - 2\sigma\varphi + \theta_2]$. Using the parameter values given in Fig.2, we can evaluate that $\varphi \approx \pi/6$, and $\theta_2 = \theta_3 = -\frac{\pi}{2}$ at the point of $\omega = 0$. Thus, at $\phi = 0$ we have $\Delta\phi_{2\uparrow} = \pi/6$ and $\Delta\phi_{2\downarrow} = 5\pi/6$, which clearly proves that the quantum interference between $\tau_{1\uparrow,2\uparrow}^{(1)}$ and $\tau_{1\uparrow,2\uparrow}^{(2)}$ (electron with spin up) is constructive. But $\tau_{1\downarrow,2\downarrow}^{(1)}$ and $\tau_{1\downarrow,2\downarrow}^{(2)}$ of spin down electron are of destructive interference. Moreover, we can get $\Delta\phi_{3\uparrow} = 5\pi/6$ and $\Delta\phi_{3\downarrow} = \pi/6$. This indicates that the situation of the quantum interference between $\tau_{1\uparrow,3\uparrow}^{(1)}$ and $\tau_{1\uparrow,3\uparrow}^{(2)}$ is just opposite to that between $\tau_{1\downarrow,2\downarrow}^{(1)}$ and $\tau_{1\downarrow,2\downarrow}^{(2)}$. In the case of $\phi = \frac{1}{2}\pi$, by a simple evaluation we find that $\Delta\phi_{2\uparrow/\downarrow} = (+/-)2\pi/3$ and $\Delta\phi_{3\uparrow/\downarrow} = (-/+)\pi/3$. Accordingly, $T_{1\sigma,j\sigma}$ does not depend on the spin index sensitively. But the constructive interference leads to the nontrivial increase of $T_{1\sigma,3\sigma}$, in comparison with $T_{1\sigma,2\sigma}$. Up to now, the characteristics of the transmission functions as shown in Fig.2, hence the tunability of charge and spin currents, have been clearly explained by analyzing the quantum interference between two kinds of paths via two different arms of the QD ring. In the case of zero magnetic field, the fact that the charge current is irrelevant to the reversal of the bias voltage can also be understood, since the profile of $T_{1\sigma,2\sigma}$ is the same as that of $T_{1\bar{\sigma},3\bar{\sigma}}$, as shown in Fig.2(a).

Now let us see what happens when lead-3 is removed from the QD-ring. At $\Gamma_3 = 0$, g_3 blows up. This leads to $|\tau_{1\sigma,2\sigma}^{(1)}| \ll |\tau_{1\sigma,2\sigma}^{(2)}|$, which implies that QD-3 provides a sharply resonant path for electron transmission. As a result, the conductance is mainly determined by $\tau_{1\sigma,2\sigma}^{(2)}$. The other path $\tau_{1\sigma,2\sigma}^{(1)}$ is only the trivial perturbation, and no spin polarization comes up. In the absence of magnetic field, $|\tau_{1\sigma,2\sigma}^{(1)}|$ is not relevant to the electron spin. Therefore, in Fig.1(d) we obtain the vanishing spin current.

In summary, in the present triple-QD ring, the local Rashba interaction provides a spin-dependent A-B phase difference. The three-terminal configuration balances the electron transmission probabilities via two different arms of the QD ring. The variation of the magnetic field strength and the QD level can adjust the phase difference between the two kinds of Feynman paths on an equal footing. Thus the spin-dependence of the electron transmission probability can be controlled by altering the exerted magnetic field or the QD levels. Furthermore, with a specific bias it is possible to obtain the tunable charge and spin currents. Before ending our work, we should remark briefly on the effect of the electron interaction which we have ignored. The electron interaction can cause the correlated electron transport, such as the Kondo effect.[18] However, our results are obtained away from the Kondo regime. To incorporate the Hubbard term into the Hamiltonian, and by using the second-order approximation to truncate the Green function equation, we have calculated the spectra of the charge and spin currents. We find that although the Hubbard U splits QD levels, hence the resonant peaks are divided into two groups, the tunable charge and spin currents remain.

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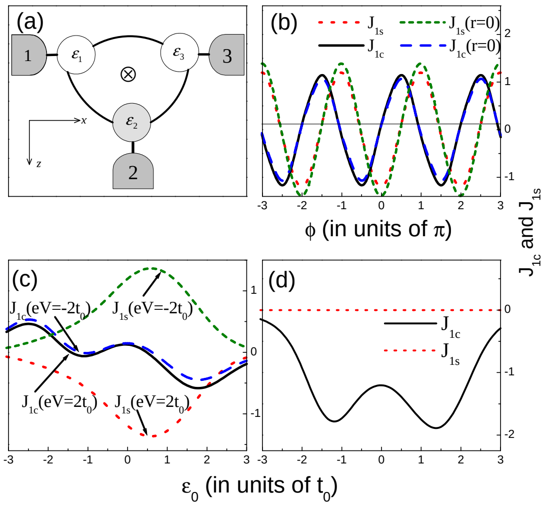

FIG. 1: (Color online) (a) A schematic of a three-terminal triple-QD ring structure with the local Rashba SO interaction on QD-2. (b)-(c) The currents versus magnetic phase factor ϕ(b) as well as the QD level ε_0(c), respectively. $\Gamma_j = 2t_0$, $\tilde{\alpha} = 0.5$. In (b) $\varepsilon_j = 0$ and in (c) $\varepsilon_j = \varepsilon_0$, $\phi = 0$. The currents without spin flip terms are shown in (b) for comparison. (d) The currents versus ε_0 in the two-terminal case($\Gamma_3 = 0$).

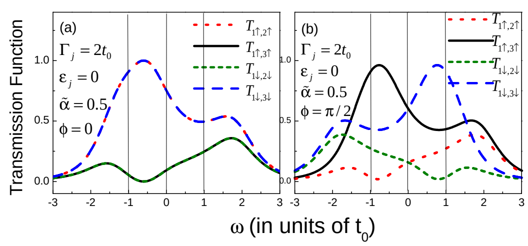

FIG. 2: (Color online) The spectra of transmission functions $T_{1\sigma,j\sigma}(j = 2, 3)$. In (a) no magnetic field is taken into account; In (b) magnetic field is considered with $\phi = 0.5\pi$.